

## IMPLICATIONS OF SUPERSYMMETRIC GRAND UNIFICATION\*

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Unification of the interactions of the Standard Model is possible in its simplest supersymmetric extension. The implications of the  $\lambda_t$  fixed-point solution on the top mass and on Higgs phenomenology is discussed. Expected correlations between the masses of various supersymmetric particles are detailed.

**1. Introduction**

The search for symmetries beyond those in the Standard Model is a constant task in modern particle physics. Since there is no compelling disagreement between the Standard Model and experiment, why then should one look for the physics beyond the Standard Model? The compelling reason is that the mechanism behind electroweak symmetry breaking is completely unknown.

The attempts to describe the electroweak symmetry breaking of the Standard Model fall largely into two broad classes: a weakly-interacting symmetry breaking sector and a strongly-interacting symmetry breaking sector. What is a natural value for the mass of the Higgs bosons that characterize the first case? It is easy to describe the requirements on the Higgs sector in the minimal version of the supersymmetric standard model, commonly referred to by its acronym MSSM. In this case the constraints from supersymmetry on the Higgs sector leads to an upper bound on the mass of the lightest physical Higgs boson.

The improvement in the precision data from LEP calls for a reevaluation of the viability of grand unified theories in the context of supersymmetry. Research has concentrated recently on including two-loop contributions in the renormalization group equations, the investigation of the impact of threshold corrections at both the electroweak and grand unified scales, and the estimation of the effects of non-renormalizable operators at the GUT scale.

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\*Talk presented by V. Barger at the *International Conference on Unified Symmetry — in the Small and the Large*, Coral Gables, Florida, January 27–30, 1994.

## 2. Phenomenological Motivations for Supersymmetry[1]

The major motivations for supersymmetry are the following

- Unification of couplings[2] — With SUSY, couplings evolve to an intersection at  $M \sim 10^{16}$  GeV. In the standard model, the gauge coupling “triangle” fails to close and unification of gauge couplings cannot be rescued even by large threshold corrections. See Figures 1a, b.
- The problems with technicolor — The problems that flavor changing neutral currents (FCNCs) and the generation of fermion masses pose for technicolor theories are well known. The simplest technicolor theories have problems when confronted with precision measurements of radiative corrections in the electroweak theory.
- Dark Matter — A candidate for cold dark matter of the Universe arises naturally in supersymmetry. The lightest supersymmetric particle (LSP) is absolutely stable if a certain symmetry (known as R-parity) exists.
- Radiative Breaking of the electroweak symmetry[3]–[14] — The Higgs mechanism can be understood in the context of supersymmetric GUTs as a negative contribution to a Higgs mass-squared by a large logarithm of the ratio of the GUT to electroweak scales.
- Proton Decay[15,16] — In the context of grand unified theories the heavy states mediate transitions between quarks and leptons, thus violating lepton and baryon number conservation. Since the rates for these transitions are governed in part by the mass of the GUT scale states, the sensitive searches for proton decay can impose severe restrictions on GUT models. In fact the minimal SU(5) model predicts proton decay at a rate already excluded by experiment. The supersymmetric models have a higher unification scale and the dimension six operators that plague the non-supersymmetric models are suppressed, but the situation is complicated by the introduction of dimension five operators in supersymmetry.

## 3. Evolution of Couplings: RGE

The unification of gauge groups is not a radical idea. In fact this idea has already been partially realized in nature as the Standard Model. The only radical idea introduced by many grand unified theories is that there is a “desert” from the electroweak scale upward to almost the Planck scale.

The gauge couplings evolve according to ordinary differential equations derived from renormalization group ideas. Large logarithms which depend on the scale of a process can be absorbed into the gauge couplings. This gives rise to a “running” gauge

coupling that depends on scale. At the one-loop level these equations are not coupled to each other, and the solutions for the reciprocal of the parameters  $\alpha_i \equiv g_i^2/4\pi$  are just linear functions of  $t = \ln(Q/M_G)$  where  $Q$  is the running mass scale and  $M_G$  is the GUT unification mass,

$$\alpha_i^{-1}(Q) = \alpha_i^{-1}(M_G) - \frac{b_i}{2\pi} t. \quad (1)$$

At the two-loop level the gauge couplings obey the RGE[17,18,19],

$$\frac{dg_i}{dt} = \frac{g_i}{16\pi^2} \left[ b_i g_i^2 + \frac{1}{16\pi^2} \left( \sum_{j=1}^3 b_{ij} g_i^2 g_j^2 - \sum_{j=t,b,\tau} a_{ij} g_i^2 \lambda_j^2 \right) \right], \quad (2)$$

The quantities  $b_i$ ,  $b_{ij}$ , and  $a_{ij}$  are determined by the particle content in the effective theory.

Although unification can be restored in the non-supersymmetric case by adding extra Higgs doublets that change the evolution of the electroweak gauge couplings  $\alpha_1$  and  $\alpha_2$ , this also lowers the scale  $M_G$  at which unification occurs and thereby exacerbates the violation of the proton decay bound rather than solving it as in the supersymmetric case. Another possibility is to put an intermediate scale between the GUT scale and the electroweak scale. Then gauge coupling unification occurs at the expense of adding new physics at this intermediate scale, thereby adding to the complexity of unification and making the resulting theory less predictive.

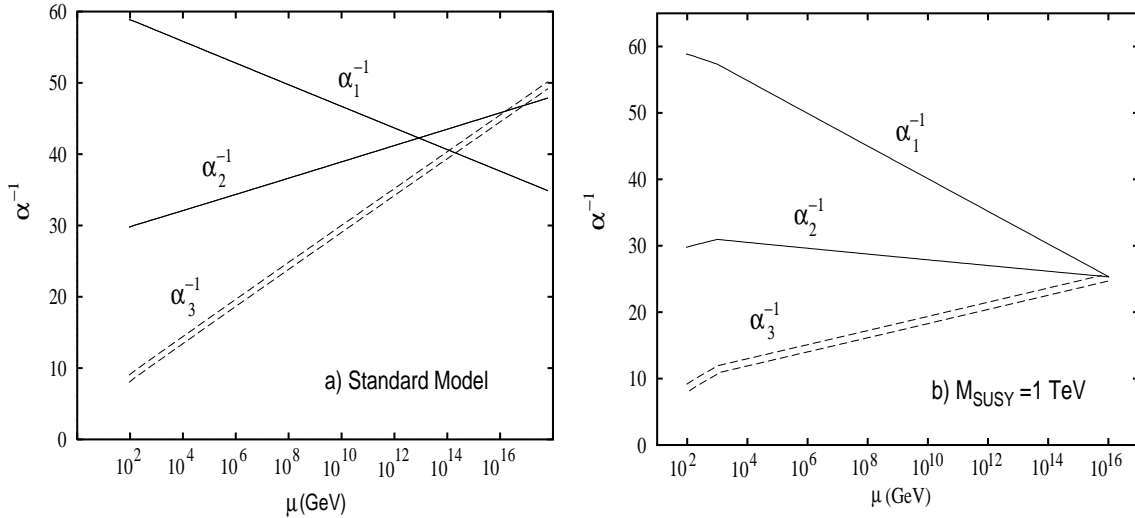


Fig. 1. Gauge coupling evolution (a) in the SM and (b) in a SUSY-GUT example.

#### 4. Yukawa Coupling Evolution and the $\lambda_t$ Fixed-Point Solution

The relationship between  $m_b$  and  $m_\tau$  in grand unified theories incorporating a desert is the most generic example of Yukawa coupling evolution. It is interesting

to note that this relationship can be made to work in supersymmetric grand unified theories, but it implies a strong constraint on the parameter space. In particular it gives a relationship between the top quark mass and the angle  $\beta$  that describes the alignment of the vacuum in two Higgs doublet models (and the MSSM).

The Yukawa couplings are related to the fermions masses in our convention[20] by

$$\lambda_b(m_t) = \frac{\sqrt{2} m_b(m_b)}{\eta_b v \cos \beta}, \quad \lambda_\tau(m_t) = \frac{\sqrt{2} m_\tau(m_\tau)}{\eta_\tau v \cos \beta}, \quad \lambda_t(m_t) = \frac{\sqrt{2} m_t(m_t)}{v \sin \beta}. \quad (3)$$

The scaling factors  $\eta_b$  and  $\eta_\tau$  relate the Yukawa couplings to their values at the scale  $m_t$ . The evolution of these Yukawa couplings is described by the RGEs,

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{16\pi^2} \left[ -\frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 + 6\lambda_t^2 + \lambda_b^2 \right], \quad (4)$$

$$\frac{dR_{b/\tau}}{dt} = \frac{R_{b/\tau}}{16\pi^2} \left[ \frac{4}{3}g_1^2 - \frac{16}{3}g_3^2 + \lambda_t^2 + 3\lambda_b^2 - 3\lambda_\tau^2 \right]. \quad (5)$$

where the ratio  $R_{b/\tau} \equiv \frac{\lambda_b}{\lambda_\tau}$ . A well-known prediction of many GUT theories is that  $R_{b/\tau}$  is equal to unity at the GUT scale[21] when the  $b$  and  $\tau$  are in the same representation of the GUT gauge group. Figures 2 and 3 show the solution of these renormalization group equations for values of the bottom quark mass. One sees that the top Yukawa coupling tends to be driven to its infrared fixed point[20],[22]–[29].

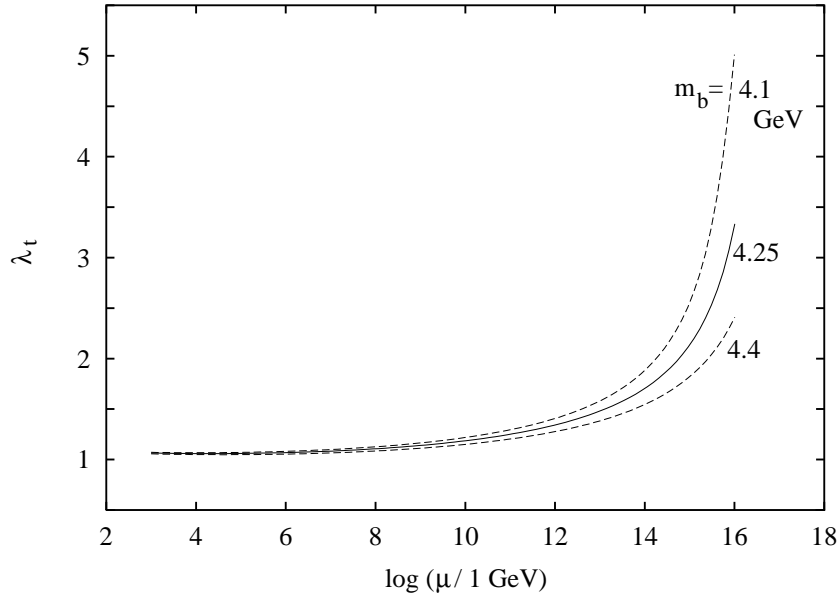


Fig. 2. If  $\lambda_t$  is large at  $M_G$ , then the renormalization group equation causes  $\lambda_t(Q)$  to evolve rapidly towards an infrared fixed point as  $Q \rightarrow m_t$  (from Ref. [20]).

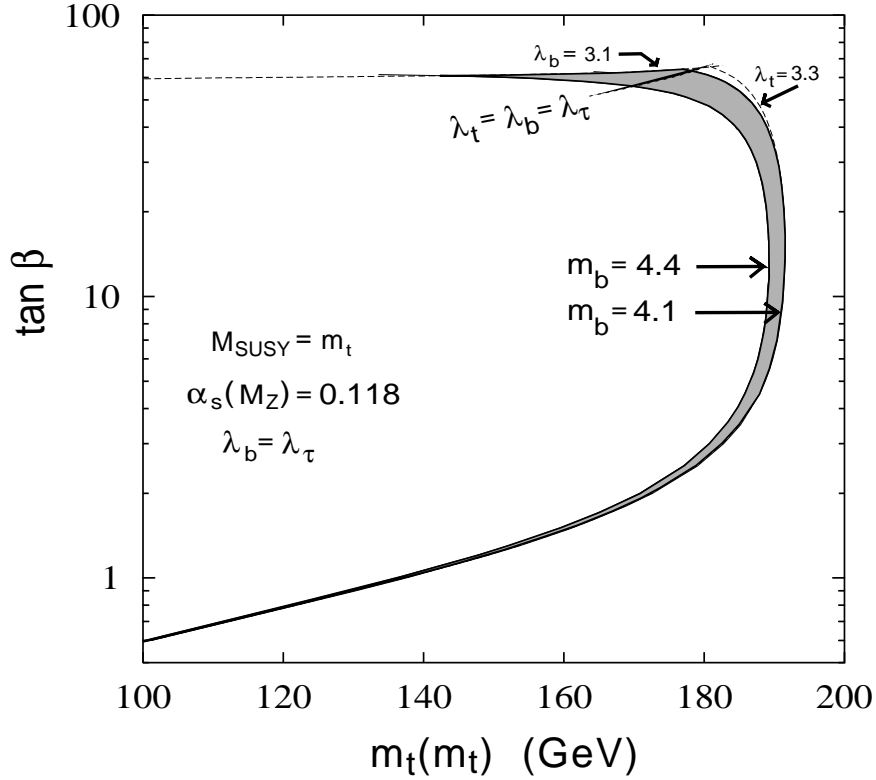


Fig. 3. Contours of constant  $m_b(m_b)$  in the  $m_t(m_t), \tan \beta$  plane (from Ref. [20]).

The fixed-point solution leads to the following relation between the top quark mass (in the  $\overline{DR}$  dimensional reduction scheme[30] with minimal subtraction) and  $\tan \beta$ .

$$\lambda_t(m_t) = \frac{\sqrt{2}m_t(m_t)}{v \sin \beta} \Rightarrow m_t(m_t) \approx \frac{v}{\sqrt{2}} \sin \beta = (192\text{GeV}) \sin \beta \quad (6)$$

Converting this relation to the top quark pole mass yields[20]

$$m_t^{\text{pole}} \approx (200\text{GeV}) \sin \beta. \quad (7)$$

If one takes the  $\lambda_t$  fixed-point solution and also assumes that the top quark mass  $m_t^{\text{pole}}$  is less than about 160 GeV, important consequences result for the Higgs sector of the MSSM. From Fig. 4 it is clear that given these assumptions  $\tan \beta$  is very near one. Since  $\tan \beta = 1$  is a flat direction in the Higgs potential, for which the associated Higgs boson is massless at tree level, and the true mass of the lightest Higgs is given almost entirely by the one-loop radiative corrections,  $m_h$  tends to be at the light end of its range. This case was discussed in detail by Diaz and Haber[31]. In this low  $\tan \beta$  region the Higgs mass is particularly sensitive to higher order corrections[32]–[34]. The upper bound on  $m_h$  that results is shown by the boundary of the theoretically disallowed region in Fig. 5.

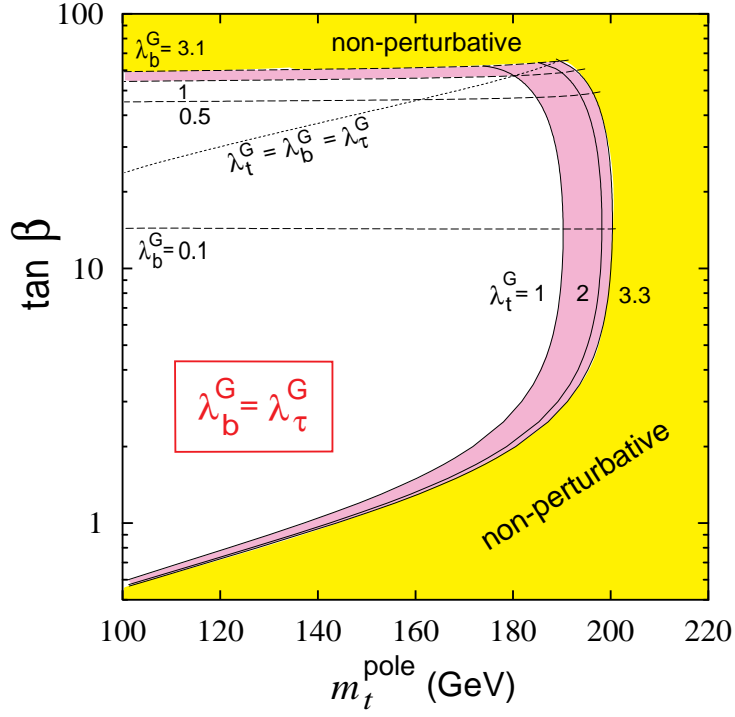


Fig. 4. The fixed-point regions are given by Yukawa couplings at the GUT scale being larger than about 1 ( $\lambda_i^G \gtrsim 1$ ). Even larger values of the Yukawa couplings results in a breakdown of perturbation theory.

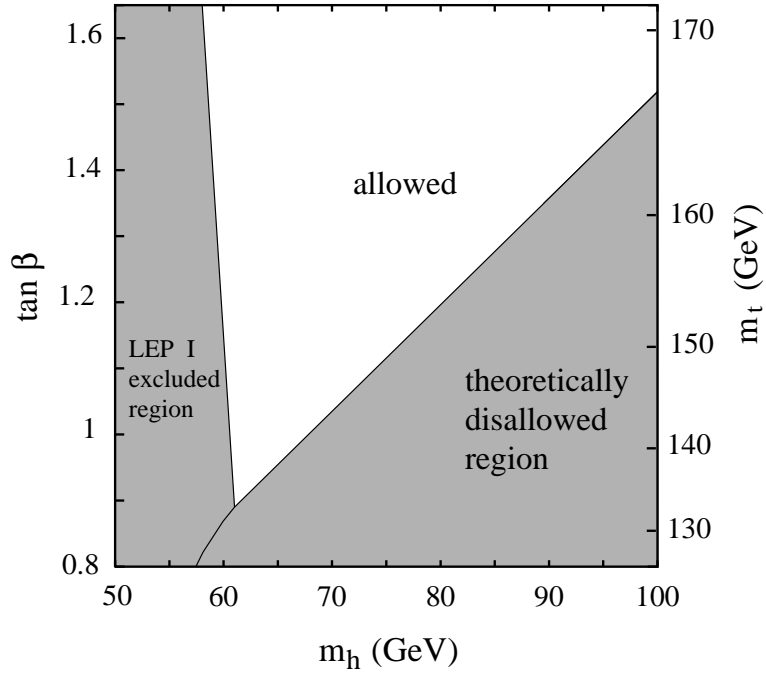


Fig. 5. The  $\lambda_t$ -fixed-point solution regions allowed by the LEP I data in the  $m_h, \tan \beta$  plane[27]. The top quark masses on the right hand side are  $m_t(\text{pole})$ , correlated to  $\tan \beta$  by the fixed-point solution.

If the top quark is sufficiently light, the fixed-point solution dictates that the Higgs potential is such that the SUSY Higgs ( $h, H, A, H^\pm$ ) searches are much more constrained. The Higgs searches at LEP have excluded a Standard Model Higgs boson with mass  $m_{H_{SM}}$  less than 62.5 GeV. The MSSM couplings give the relation

$$\Gamma(Z \rightarrow Z^* h) = \sin^2(\beta - \alpha) \Gamma(Z \rightarrow Z^* H_{SM}) . \quad (8)$$

for  $m_h = m_{H_{SM}}$ . Here the angle  $\alpha$  describes the mixing between the CP-even components of the two Higgs doublets. For a top quark mass less than about 170 GeV, the fixed point gives  $\tan \beta \approx 1$ . Then in this region of parameter space

$$\sin^2(\beta - \alpha) \sim 1$$

so that the bound on the MSSM lightest Higgs is close to the bound on the Standard Model Higgs.

## 5. Threshold Corrections

As discussed above, the requirement of gauge and Yukawa coupling unification can be successful in the minimal versions of supersymmetric grand unified theories and can result in strong constraints on the parameter space of the model. With the improved electroweak data recently made available, a number of groups have been attempting to incrementally refine the theoretical predictions. The one-loop renormalization group equations have been extended to two-loops in the Yukawa sector[18,20] and in the soft-supersymmetry breaking parameters[35]. Moreover threshold corrections at the GUT and electroweak (and electroweak-scale SUSY thresholds) have been investigated[26,28,34,36,37,38,39,41]. The RGE evolution yields the generic features that result from the large separation of scales typical in GUT theories. Threshold effects are typically model-dependent and sensitive to the detailed spectrum (the supersymmetric spectrum, the top mass, etc. at the electroweak scale and the superheavy spectrum at the GUT scale)<sup>†</sup>. Threshold effects have been incorporated into the predictions from gauge and Yukawa coupling unification and in the Higgs potential, and eventually will be included in the full supersymmetric spectrum as well[42].

The theory below the grand unified theory is an effective theory with the heavy GUT-scale particles integrated out. Since the heavy particles do not completely fill the representations of the grand unified gauge group, the group is broken and the RGEs of the effective symmetry exhibit this broken symmetry and the gauge couplings diverge below the GUT scale. The process of integrating out the heavy particles gives rise to threshold corrections that depend on the details of the grand unified theory. Since the threshold corrections at the GUT scale depend on the superheavy spectrum, one

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<sup>†</sup>The Yukawa coupling unification condition  $\lambda_b^G = \lambda_\tau^G$  is itself a model-dependent feature satisfied in simple GUT scenarios.

therefore expects these corrections to be constrained by the proton decay limits. In a similar way there are threshold corrections at the electroweak scale from the effective theories that are introduced there (the scale of supersymmetry can be chosen to be different than the electroweak scale). Typically one makes some assumptions about the supersymmetric spectrum to simplify the problem (such as in a supergravity-based scenario). Ultimately one hopes that these model-dependent features can be used to distinguish between the various realization of the supersymmetric models.

The threshold corrections to Yukawa coupling unification are also relevant[41] to the analyses of GUT scale mass matrix ansätze. Relations between fermion masses and mixing angles that arise in these scenarios will be modified by model-dependent effects.

## 6. Where are the Sparticles?

Sometimes supersymmetry is criticized because of a proliferation of parameters. This is not necessarily a fair criterion, since in some minimal versions of supergravity theories the complete mass spectrum and couplings can be explained with the addition of as few as three or five parameters. At the present time our ignorance of the mechanism of supersymmetry breaking should make us cautious about sweeping statements about the supersymmetric particles that rely on some GUT-scale assumptions; however, it is not unreasonable to expect there will be correlations between the supersymmetric particle masses and couplings since we hope that the ultimate theory that describes them near the Planck scale is a simple and economical one. Figure 6 shows representative results for RGE evolution of the sparticle masses.

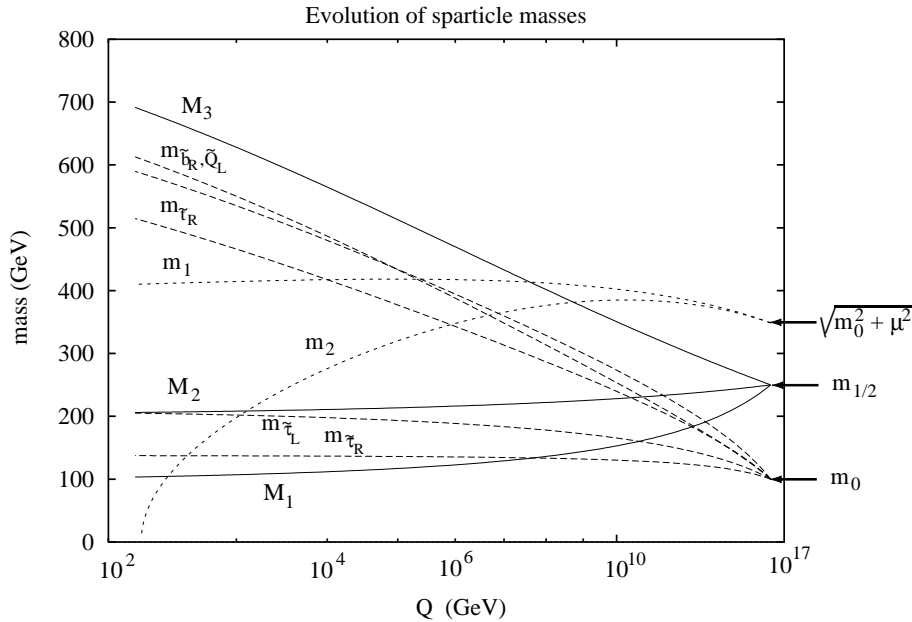




Fig. 6. The evolution of the sparticle spectrum from the unification scale down to the electroweak scale. The characteristic behavior exhibited by the mass parameters are typical of renormalization group equation evolution.

A popular and convenient approach[43] to obtaining a solution to the soft-supersymmetry breaking RGEs is to define some inputs at the GUT scale and some inputs at the electroweak scale. We have dubbed this approach the ambidextrous approach[13] to distinguish it from the bottom-up[9] and top-down[44] approaches where all inputs are defined at the same scale. Common to all of these approaches is the requirement that correct electroweak symmetry breaking (EWSB) be achieved. This is accomplished by imposing two minimization conditions obtained by minimizing the Higgs potential.

The tree-level Higgs potential is given by

$$V_0 = (m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2 + m_3^2(\epsilon_{ij}H_1^iH_2^j + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)[|H_1|^2 - |H_2|^2]^2 + \frac{1}{2}g^2|H_1^{i*}H_2^i|^2, \quad (9)$$

where  $m_{H_1}$ ,  $m_{H_2}$ , and  $m_3$  are soft-supersymmetry breaking parameters and  $\epsilon_{ij}$  is the total antisymmetric tensor. The minimum of the Higgs potential must occur by the acquisition of vacuum expectation values. Minimizing  $V_0$  with respect to the two neutral CP-even Higgs degrees of freedom yields

$$\frac{1}{2}M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2. \quad (10)$$

$$-B\mu = \frac{1}{2}(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2) \sin 2\beta. \quad (11)$$

The masses in these equations are running masses that depend on the scale  $Q$  in the RGEs that describe their evolution. Hence the solutions obtained are functions of the scale  $Q$ . Equations (10) and (11) are a particularly convenient form since the gauge couplings dependence (the D-terms in the language of supersymmetry) is isolated in Eq. (10). This equation also clearly shows the fine-tuning problem that may be present in the radiative breaking of the electroweak symmetry. For large values of  $|\mu|$ , there must be a cancellation between large terms on the right hand side to obtain the correct experimentally measured  $M_Z$  (or equivalently the electroweak scale). For  $\tan \beta$  near one, a cancellation of large terms must occur. Finally, these minimization equations illustrate the power of the ambidextrous approach. For EWSB to be satisfied one need only specify  $m_t$ ,  $M_Z$ ,  $\tan \beta$  at the electroweak scale and the common gaugino mass  $m_{1/2}$ , scalar mass  $m_0$ , and the trilinear scalar coupling  $A$  at the GUT scale. Then one solves the minimization equations given above to obtain  $\mu$  (up to a sign) and  $B$ , thereby implicitly satisfying the EWSB requirement. For more details, see Ref. [13].

A heavy top quark produces large corrections to the Higgs potential of the MSSM[45]. Gamberini, Ridolfi, and Zwirner showed[5] that the tree-level Higgs potential is inadequate for the purpose of analyzing radiative breaking of the electroweak

symmetry because the tree-level Higgs vacuum expectation values  $v_1$  and  $v_2$  are very sensitive to the scale at which the renormalization group equations are evaluated. The one-loop corrections to the Higgs potential effectively moderates this sensitivity to the scale  $Q$ . The one-loop corrections are conveniently calculated using the tadpole method[13,46,47]. The corrections to Eqs. (10) and (11) can be obtained by calculating the two tadpoles with two independent CP-even Higgs as external lines as in Fig. 7. The one-loop corrected minimization conditions can then be used to generate a complete supersymmetric particle spectrum which satisfies EWSB.

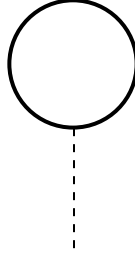


Fig. 7. The tadpole diagrams offer a simple method to obtain the one-loop modifications to the minimization conditions. The loop consists of all matter and gauge-Higgs contributions, and the external lines are the two CP-even Higgs fields.

Including only the leading contribution coming from the top quark loop (and neglecting the D-term contributions to the squark masses) one obtains the expressions

$$\frac{1}{2}M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 - \frac{3g^2 m_t^2}{32\pi^2 M_W^2 \cos 2\beta} \left[ 2f(m_t^2) - f(m_{t_1}^2) - f(m_{t_2}^2) + \frac{f(m_{t_1}^2) - f(m_{t_2}^2)}{m_{t_1}^2 - m_{t_2}^2} ((\mu \cot \beta)^2 - A_t^2) \right], \quad (12)$$

$$-B\mu = \frac{1}{2}(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2) \sin 2\beta - \frac{3g^2 m_t^2 \cot \beta}{32\pi^2 M_W^2} \left[ 2f(m_t^2) - f(m_{t_1}^2) - f(m_{t_2}^2) - \frac{f(m_{t_1}^2) - f(m_{t_2}^2)}{m_{t_1}^2 - m_{t_2}^2} (A_t + \mu \cot \beta)(A_t + \mu \tan \beta) \right], \quad (13)$$

where

$$f(m^2) = m^2 \left( \ln \frac{m^2}{Q^2} - 1 \right). \quad (14)$$

The extra one-loop contribution included above renders the solution less sensitive to the scale  $Q$ [8,9,10,13,48], as can be shown explicitly by examining the relevant renormalization group equations for the parameters that enter into the minimization

conditions. The complete expressions for the one-loop contributions can be found in Ref. [13]. The fine-tuning problem is alleviated somewhat, but not entirely, by the inclusion of one-loop corrections to the Higgs potential. As our naturalness criterion we require

$$|\mu(m_t)| < 500 \text{ GeV} . \quad (15)$$

The gaugino masses are related (through one-loop order) by the same ratios that describe the gauge couplings at the electroweak scale. This observation, together with the fact that  $|\mu|$  is large, yields simple correlations between the lightest chargino and neutralinos and the gluino[6,10,49], namely

$$M_{\chi_1^0} \simeq M_1 , \quad (16)$$

$$M_{\chi_1^\pm} \simeq M_{\chi_2^0} \simeq M_2 = \frac{\alpha_2}{\alpha_1} M_1 \simeq 2M_1 \simeq 2M_{\chi_1^0} , \quad (17)$$

$$m_{\tilde{g}} = M_3 = \frac{\alpha_3}{\alpha_2} M_2 = \frac{\alpha_3}{\alpha_1} M_1 . \quad (18)$$

In our analysis the quantities in these equations are evaluated at scale  $m_t$ . The heaviest chargino and the two heaviest neutralino states are primarily Higgsino with

$$M_{\chi_2^\pm} \simeq M_{\chi_3^0} \simeq M_{\chi_4^0} \simeq |\mu| . \quad (19)$$

As previously noted the mass of the lightest Higgs  $h$  arises mainly from radiative corrections[27,31,34,50]. The heavy Higgs states are (approximately) degenerate  $\approx M_A$  because at tree-level  $M_A = -\frac{B\mu}{\sin 2\beta} \approx -B\mu$  is large. The squark and slepton masses also display simple asymptotic behavior at large  $|\mu|$ . The first and second squark generations are approximately degenerate. The splitting of the stop quark masses grows as  $|\mu|$  increases. The splitting of the sbottom states does not change much with  $\mu$  for small  $\tan\beta$ .

The approximate experimental bounds that we impose are listed in Table 1. Together with our naturalness criteria  $|\mu(m_t)| < 500 \text{ GeV}$ , these bounds give the allowed region in the  $m_0, m_{1/2}$  plane shown as the shaded areas in Fig. 8.

Table 1. Approximate experimental bounds.

Particle	Experimental Limit (GeV)
gluino	120
squark, slepton	45
chargino	45
neutralino	20
light higgs	60

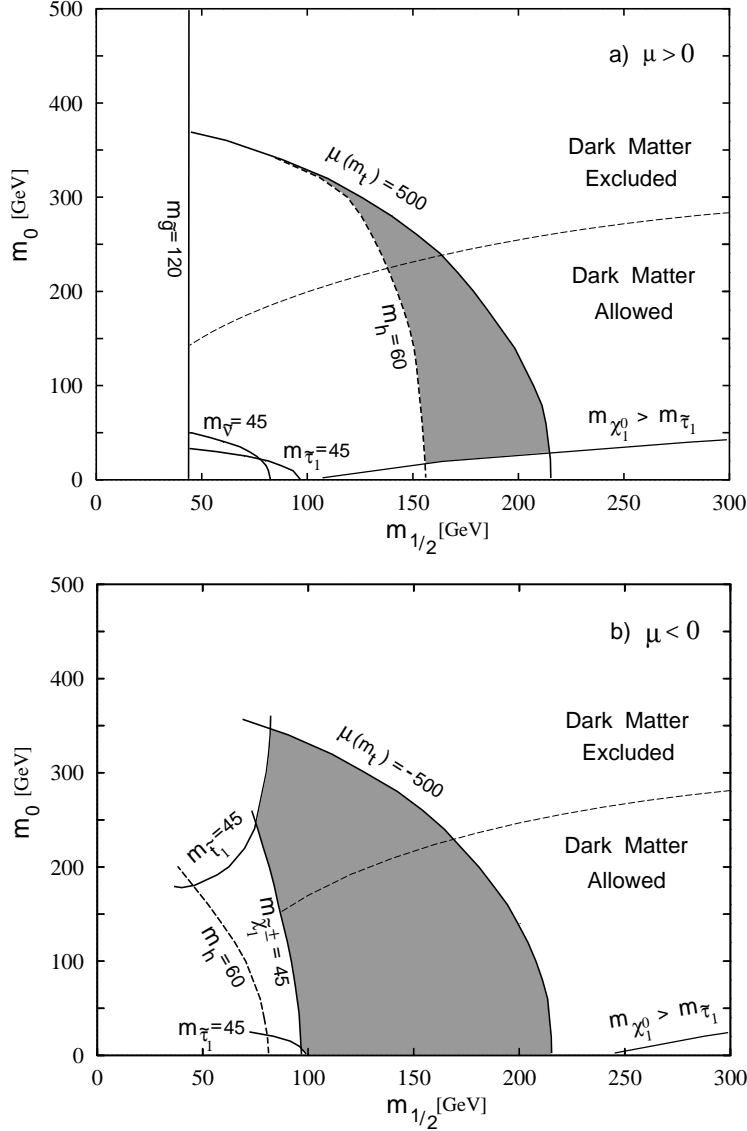


Fig. 8. Allowed regions of parameter space for  $m_t(m_t) = 160$  GeV,  $\tan \beta = 1.47$  (a low- $\tan \beta$  fixed-point solution) (from Ref. [13]).

The prediction for  $m_h$  in the low- $\tan \beta$  region is particularly sensitive to two-loop corrections[34]. Hence the precise location of the  $m_h = 60$  GeV contour is somewhat uncertain. The MSSM has conserved R-parity so the lightest supersymmetric particle (LSP) is stable. Usually the LSP is the lightest neutralino, but for small values of  $m_0$  the supersymmetric partner of the tau lepton is sometimes lighter. For the lightest SUSY particle to be neutral there is an upper bound on the value of  $m_{1/2}$  for small  $m_0$ . In particular such an upper bound exists for no-scale models ( $m_0 = 0$ ), and is more stringent for  $\mu > 0$  due to the mixing between the left and right handed  $\tilde{\tau}$ , giving a stau lighter than the lightest neutralino. The LSP can also account for the dark matter of the Universe[51,52]. The large values of  $\mu$  obtained from the low  $\tan \beta$

solution result in the lightest neutralino being predominantly gaugino (see Figure 9). This leads to a reduced rate of annihilation of neutralinos and can provide too much relic abundance and overclose the Universe. This constraint is shown as the dashed line in Figure 8; this line should be regarded as a semi-quantitative one only since the contributions of  $s$ -channel poles that can enhance the annihilation rate have been neglected.

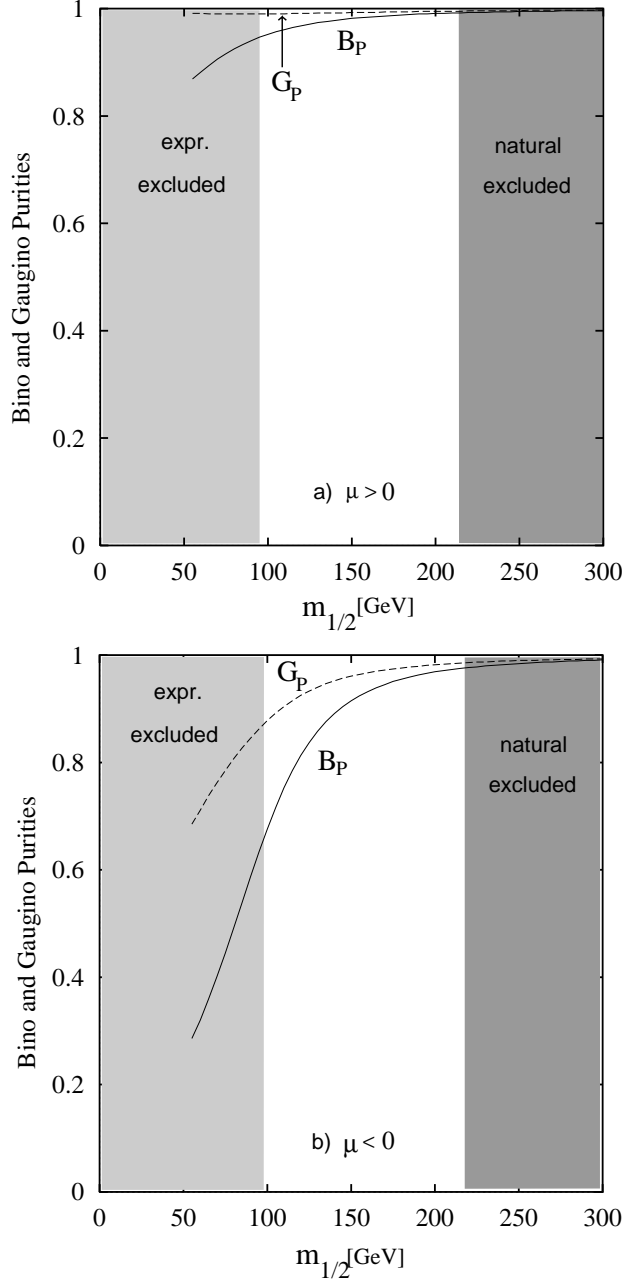


Fig. 9. Bino and gaugino purities for low  $\tan \beta$  fixed-point solution.

Figure 10 shows the dependence of the sparticle masses on the parameter  $m_{1/2}$ . The large  $|\mu|$  obtained from the low- $\tan\beta$  solutions lead to highly correlated masses. Figure 11 shows the squark masses, which are quite degenerate for the light families.

Figure 12 shows the supersymmetric particle mass dependence on the parameter  $m_0$  for a fixed value of  $m_{1/2}$ . The lighter top squark eigenstate  $\tilde{t}_1$  has an approximately constant mass with increasing  $m_0$ . This occurs because  $|\mu|$  increases with  $m_0$  giving rise to increased mixing between the left- and right-handed top squarks (lowering the lightest mass eigenstate).

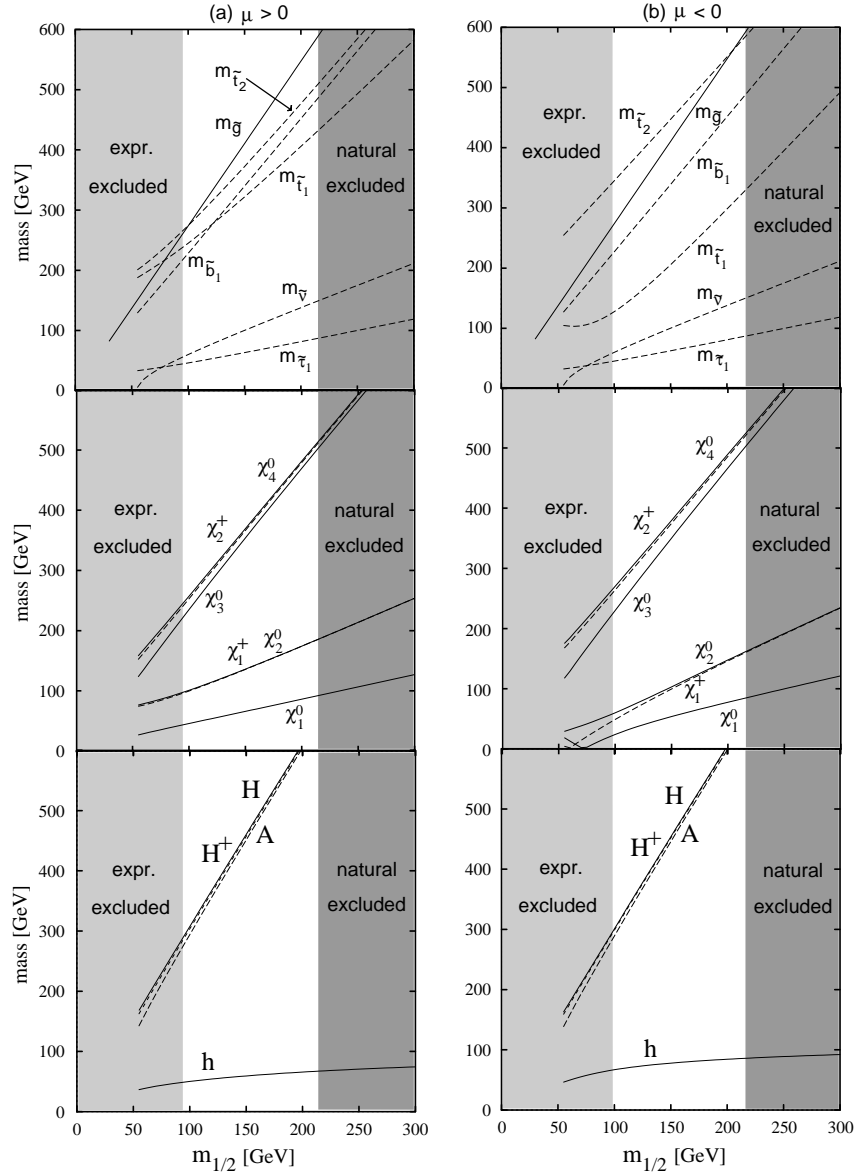


Fig. 10. Particle spectra as a function of  $m_{1/2}$ . The shaded bands at the left are regions excluded by the experimental constraints. The shaded bands at the right labeled “natural excluded” require  $|\mu(m_t)| > 500$  GeV (from Ref. [13]).

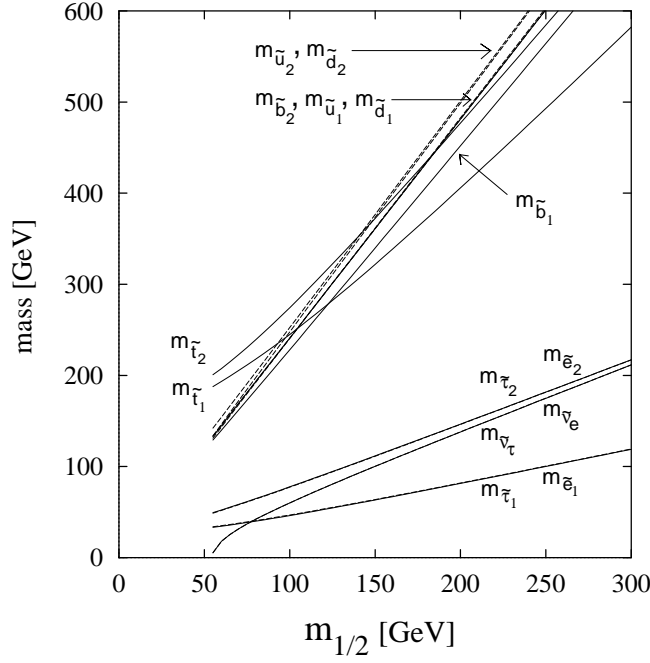


Fig. 11. Supersymmetric scalar masses showing asymptotic behavior (from Ref. [13]).

## 7. SUSY Signals

At hadron colliders a plethora of sparticle production processes are possible, as illustrated in Fig. 13. The SUSY particles must be created in pairs (for theories with an R-symmetry). Gluinos should be copiously produced at a future hadron collider. As the mass of the gluino increases, new decay channels open up. Figure 14 shows typical branching fraction for gluinos assuming that  $m_{\tilde{g}} < m_{\tilde{q}}$  [53] (in this figure it is not assumed that  $\mu$  is large as required by radiative breaking of the electroweak symmetry). The predicted cascade gluino decays provide multiple signals for experimental searches. Gluino decays in the Yukawa unified supergravity scenario with radiative electroweak symmetry breaking have been investigated in Ref. [54]. A phenomenological discussion of the Yukawa unified no-scale model can be found in Ref. [55].

## 8. Conclusions

The continued viability of supersymmetric grand unified theories with respect to the increased precision of the low energy data calls for more refined theoretical analyses. The following observations summarize the principal points of this review.

- A low-energy supersymmetry is consistent with a desert unification scenario in grand unified theories.

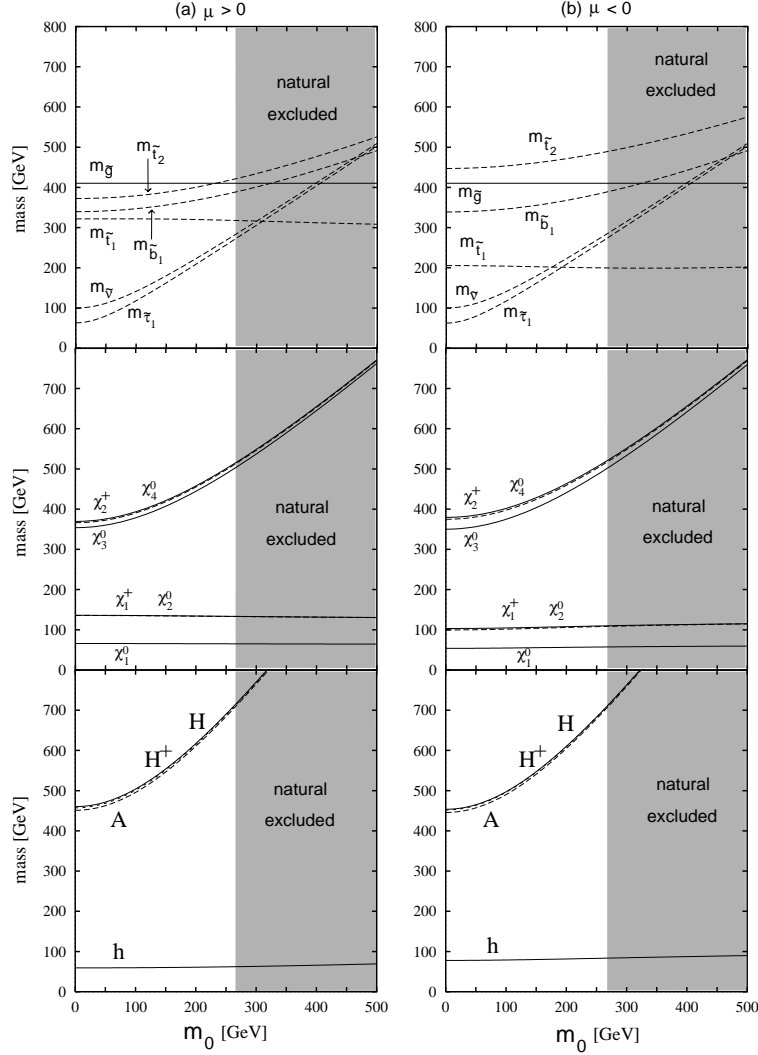


Fig. 12. The supersymmetric particle mass dependence on the parameter  $m_0$  for (a)  $\mu > 0$  and (b)  $\mu < 0$  with  $m_{1/2} = 150 \text{ GeV}$  and  $A^G = 0$  (from Ref. [13]).

- The observed ratio  $m_b/m_\tau$  is consistent with SUSY GUTs. In fact, this ratio indicates that the top quark Yukawa coupling is near its infrared fixed point; this situation has significant implications for SUSY Higgs searches if the top quark is lighter than about 165 GeV. In that case the upper bound on  $m_h$  is of order 100 GeV.
- Solutions with a  $\lambda_t$  fixed point,  $m_t \lesssim 175 \text{ GeV}$  and radiative breaking of the electroweak symmetry breaking are allowed by our naturalness criterion  $|\mu(M_Z)| \simeq |\mu(m_t)| < 500 \text{ GeV}$  for both signs of the supersymmetric Higgs mass parameter  $\mu$ . These solutions are characterized by relatively large values of  $|\mu|$ , which implies that the supersymmetric particle spectrum displays a simple asymptotic behavior in the simplest supergravity models.



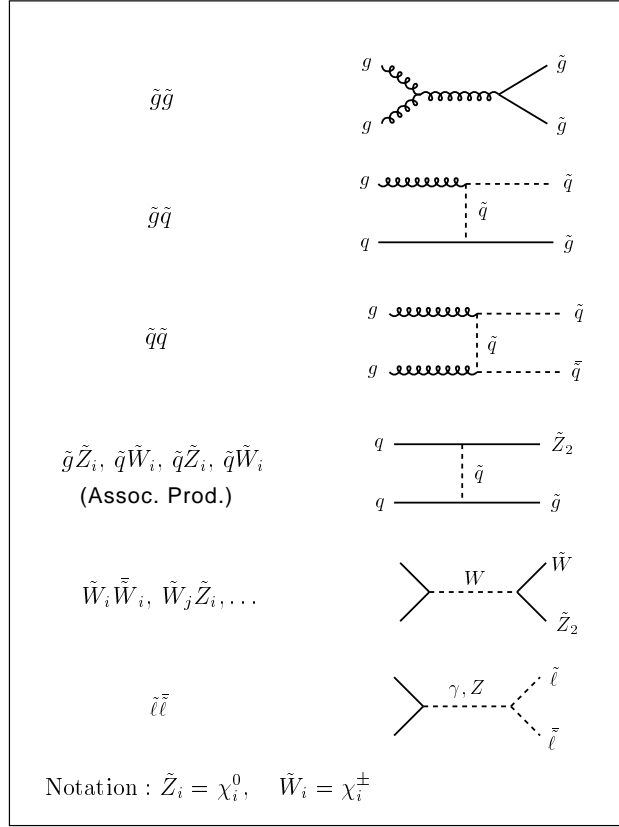


Fig. 13. SUSY production processes at hadron colliders.

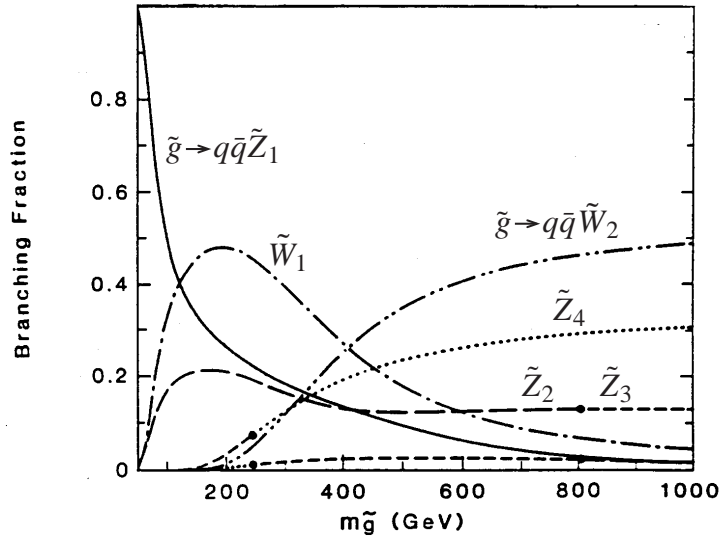


Fig. 14. Gluino branching fractions assuming that  $m_{\tilde{g}} < m_{\tilde{q}}$  (from Ref. [53]).

- In the early universe the LSP annihilates sufficiently (at least in the approximation that  $s$ -channel pole annihilation is neglected) over most of the parameter space  $m_0 \lesssim 300$  GeV, so as not to overclose the universe.
- The one-loop corrections to the Higgs potential somewhat ameliorate the fine-tuning problem.
- The tadpole method is a convenient way to calculate the one-loop minimization conditions. We have obtained these conditions in an analytic form including all contributions from the gauge-Higgs sector and matter multiplets. This method is easily extended to non-minimal Higgs sectors or to models with additional low-lying states.

## Acknowledgements

This research was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, in part by the U.S. Department of Energy under contract no. DE-AC02-76ER00881, and in part by the Texas National Laboratory Research Commission under grant nos. RGFY93-221 and FCFY9302. MSB was supported in part by an SSC Fellowship. PO was supported in part by an NSF Graduate Fellowship.

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